# PROGRESSION THROUGH WRITTEN CALCULATION

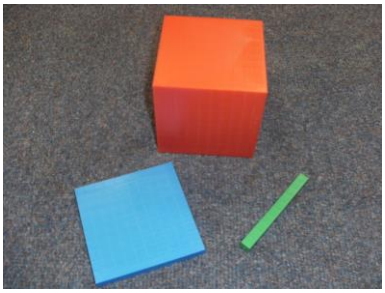
\+ - × ÷

Children will use practical resources to work mathematically which also encourages them to experiment with counting and with the number system.

Base 10

Place Value Arrow Cards

Tens and Units



Dice

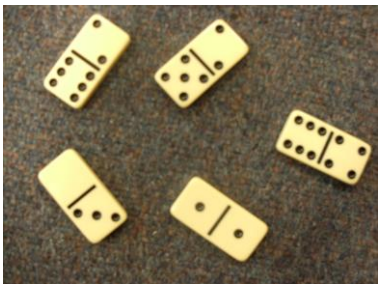
Place Value Counters

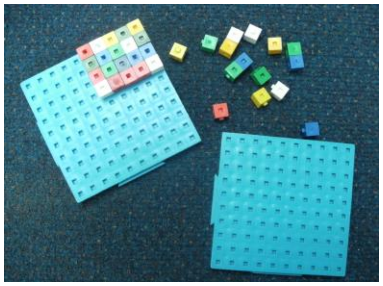
Numicon

Cuisenaire Rods

Cubes/ Blocks

Dominoes



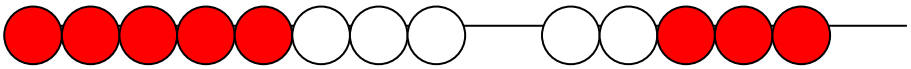


Magnets

Counters

100 Squares

Counting bears



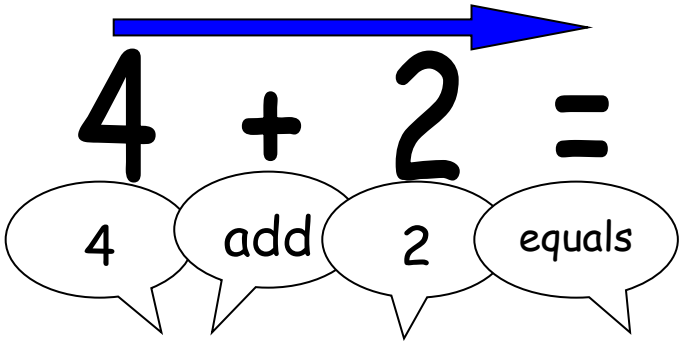
Bead strings

# PROGRESSION THROUGH WRITTEN CALCULATION

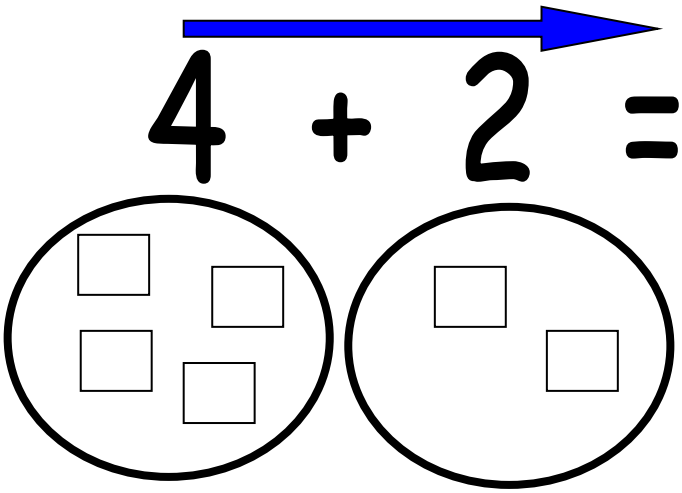
## ADDITION +

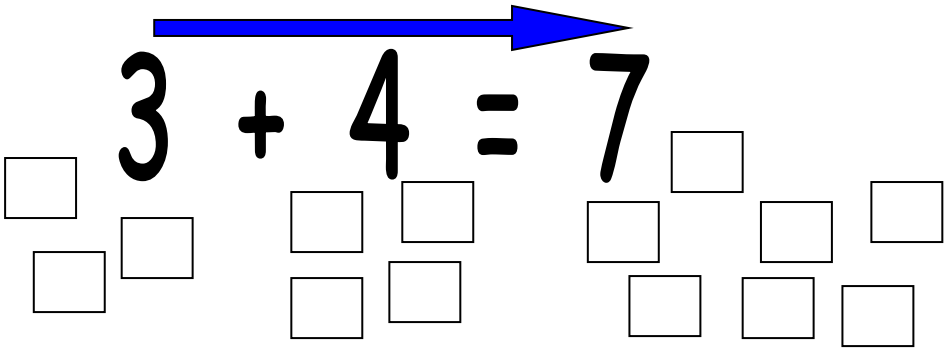
(add, addition, more, plus, increase, sum, total, altogether, equals, inverse)

### Reading the number sentence

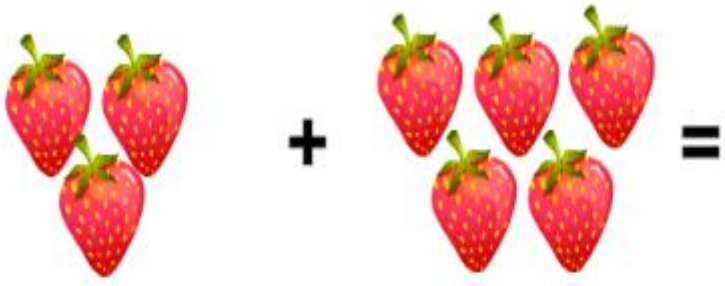


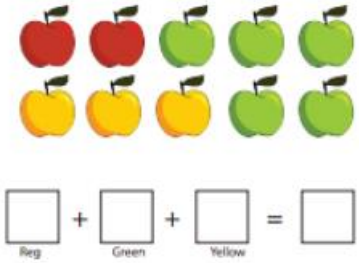

### Arranging the number sentence



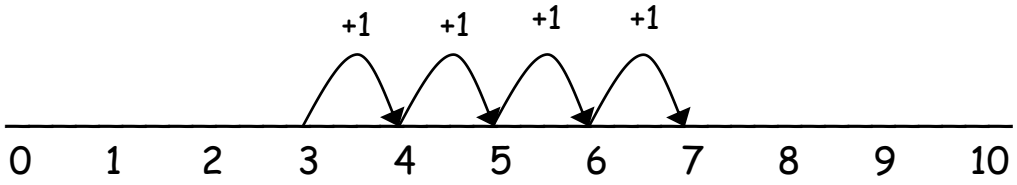
## Addition using objects/ pictorial representation



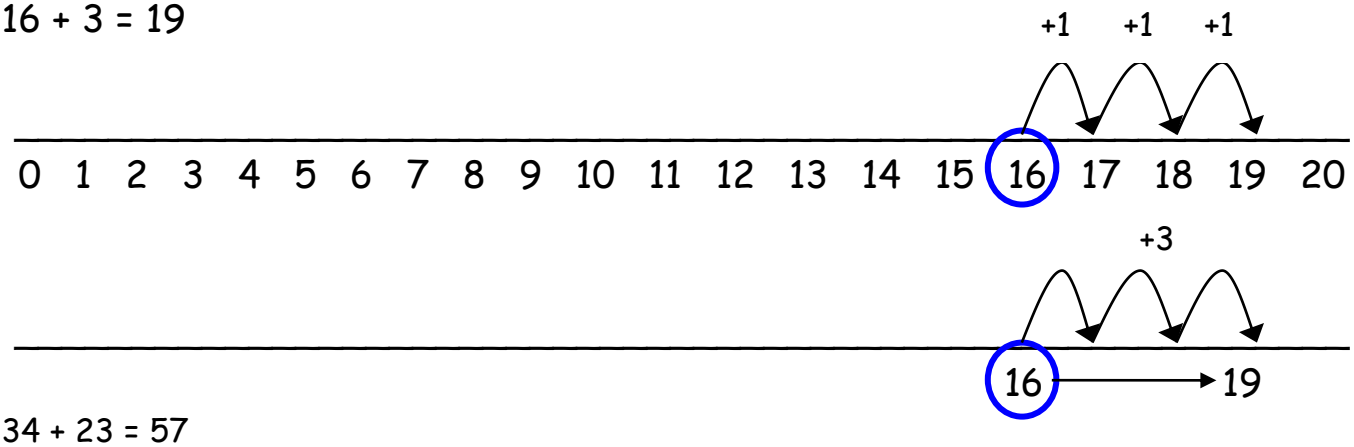



## Number Line

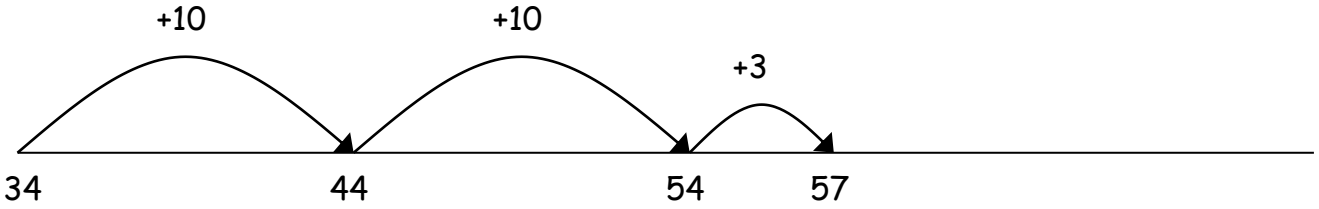
3 + 4 = 7



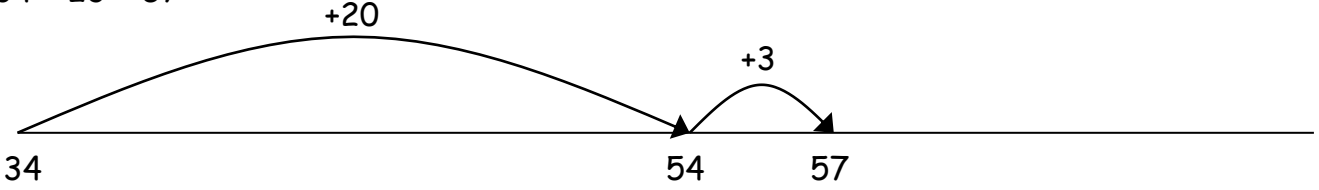

16 + 3 = 19




34 + 23 = 57

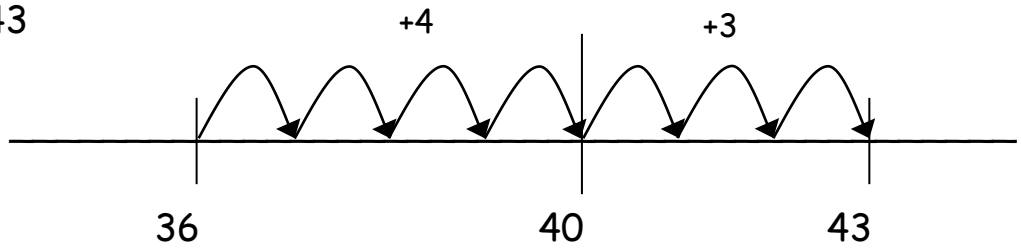



34 + 23 = 57



- Using bridging

36 + 7 = 43




## Hundred Square

28 + 1 = 29  43 + 1 = 44  86 + 1 = 87

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

40 + 10 = 50  70 + 10 = 80  23 + 10 = 33  82 + 10 = 92

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

30 + 4 = 34  50 + 6 = 56  80 + 2 = 82

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

20 + 60 = ?  Start with the larger number

So 60 + 20 = 80

## Partitioning method (addition)

# 68 + 9

**Step 1:** 8 + 9 = 17

**Step 2:** 60 + 17 = 77

# 47 + 58

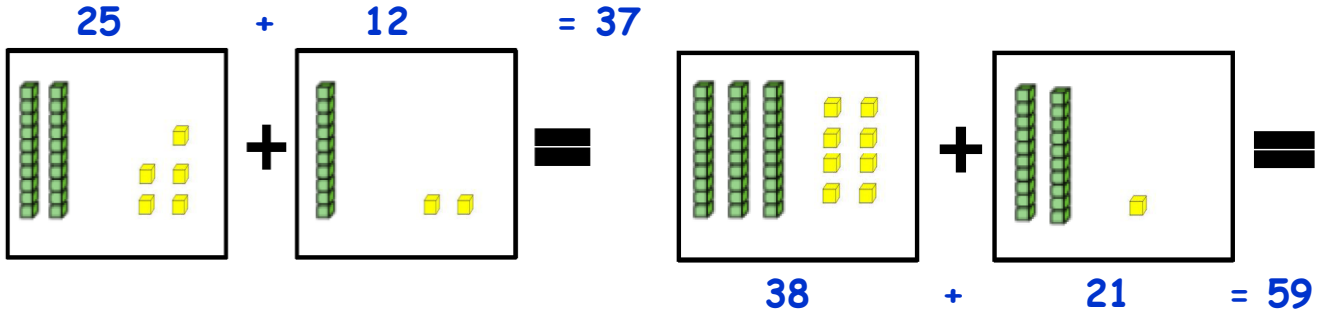
**Step 1:** 40 + 50 = 90

**Step 2:** 7 + 8 = 15

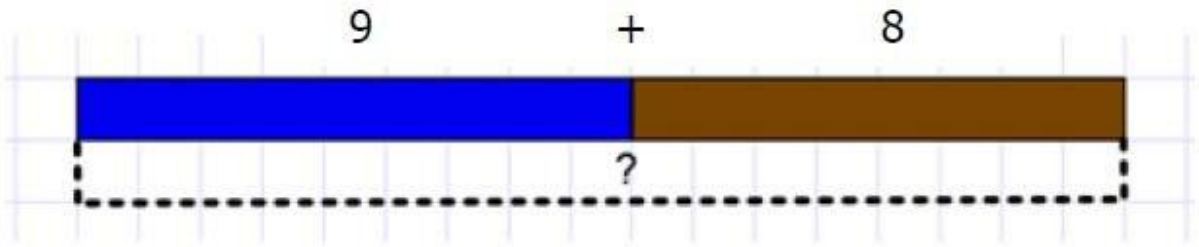
**Step 3:** 90 + 15 = 105

# 241 + 328

| | | |
|---|---|---|
| **Hundreds** | 200 + 300 | = 500 |
| **Tens** | 40 + 20 | = 60 |
| **Units** | 1 + 8 | = 9 + |
| | | 569 |




## Bar model

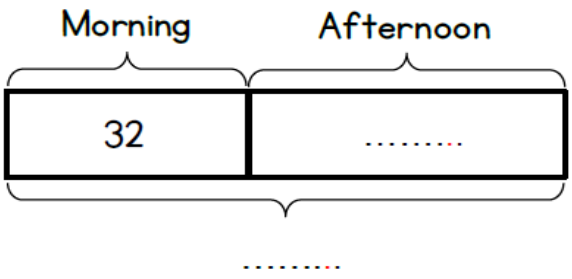



Lisa owns a coffee shop.

In the morning she sells 32 mugs of tea.

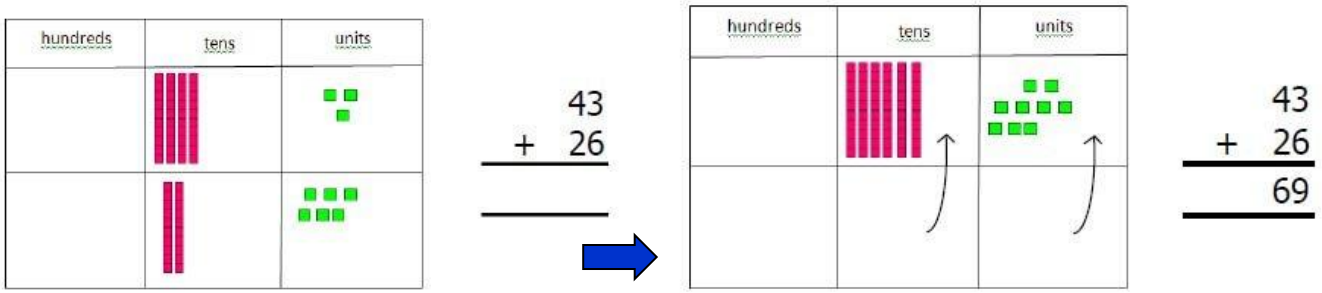
In the afternoon she sells 45 mugs of tea.

How many mugs of tea does she sell in total?

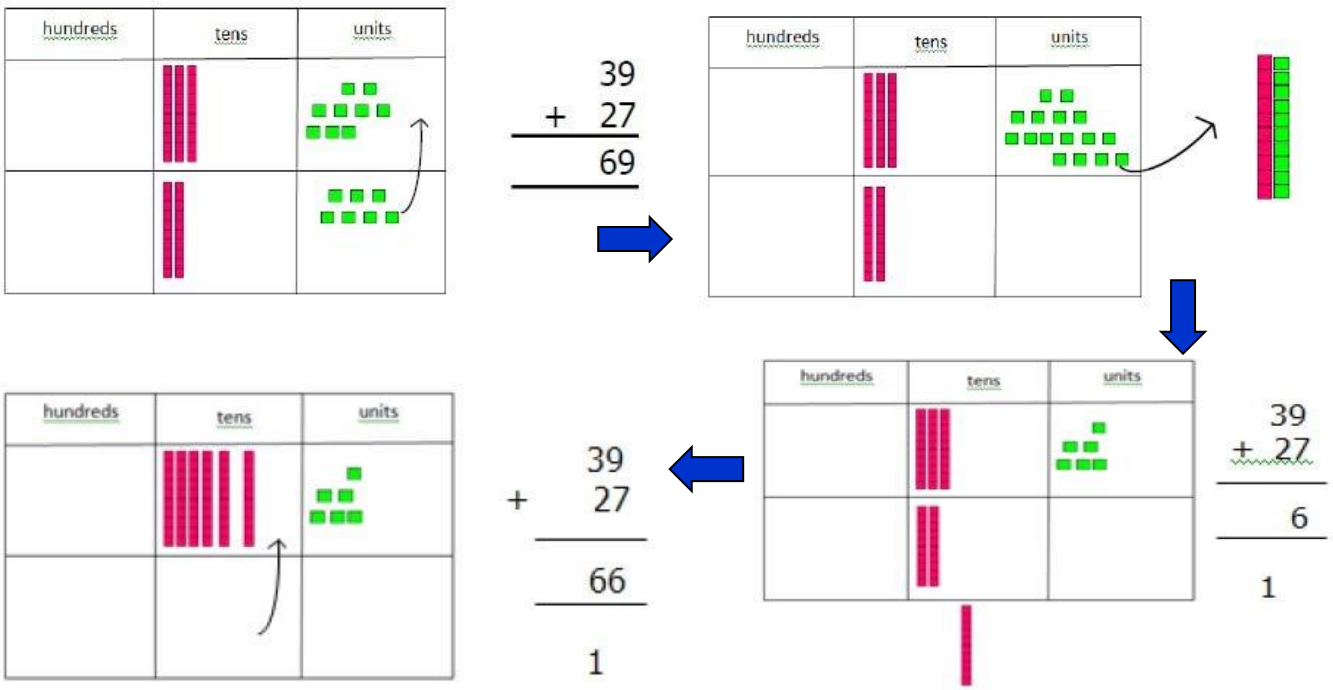


## Supported column method (addition)

*Without regrouping*




*With regrouping*




## Column method (addition)

| | | | | | | |
|---|---|---|---|---|---|---|
| 34 | 67 | 267 | 241 | 207 | 587 | 789 |
| 23+ | 24+ | 85+ | 328+ | 453+ | 375+ | 642+ |
| 57 | 91 | 352 | 569 | 660 | 962 | 1431 |
| | 1 | 1 1 | | 1 | 1 1 | 1 1 1 |

| | | |
|---|---|---|
| 42 | 6244 | 834 |
| 6432 | 8 | 92 |
| 786 | 36 | 5+ |
| 3 | 935+ | 931 |
| 4681 | 7223 | 1 1 |
| 11944 | | |

## Adding decimals

4 tenths and 3 tenths are 7 tenths

**So** ⟶ 0.4 + 0.3 = 0.7

8 tenths and 9 tenths are 17 tenths

**So** ⟶ 0.8 + 0.9 = 1.7 **not 0.17**

$$\begin{array}{r} 3.33 \\ 2.5\ + \\ \hline 5.83 \end{array} \qquad \begin{array}{r} 6.71 \\ 2.9\ + \\ \hline 9.61 \\ 1 \end{array} \qquad \begin{array}{r} 8.67 \\ 9.8\ + \\ \hline 18.47 \\ 1\ 1 \end{array}$$

£2.41 + £3.53 = £5.94

**If…**
200 + 300 = 500
40 + 50 = 90
1 + 3 = 4

$$\begin{array}{r} £2.41 \\ £3.53+ \\ \hline £5.94 \end{array}$$

**Then…**
£2.00 + £3.00 = £5.00
£0.40 + £0.50 = £0.90
£0.01 + £0.03 = £0.04
£5.00 + £0.90 + £0.04 = £5.94

£3.85 + £8.67 = £12.52

**If…**
300 + 800 = 1100
80 + 60 = 140
5 + 7 = 12

$$\begin{array}{r} £\ 3.85 \\ £\ 8.67+ \\ \hline £12.52 \\ 1\ \ 1\ \ 1 \end{array}$$

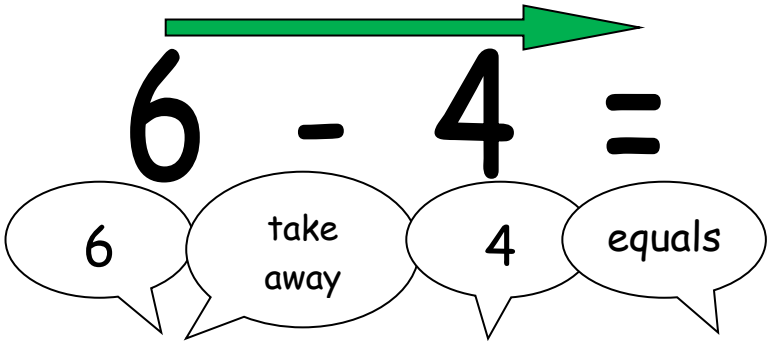
**Then…**
£3.00 + £8.00 = £11.00
0.80 + £0.60 = £1.40
0.05 + £0.07 = £0.12
£11.00 + £1.40 + £0.12 = £12.52

PROGRESSION THROUGH WRITTEN CALCULATION FOR

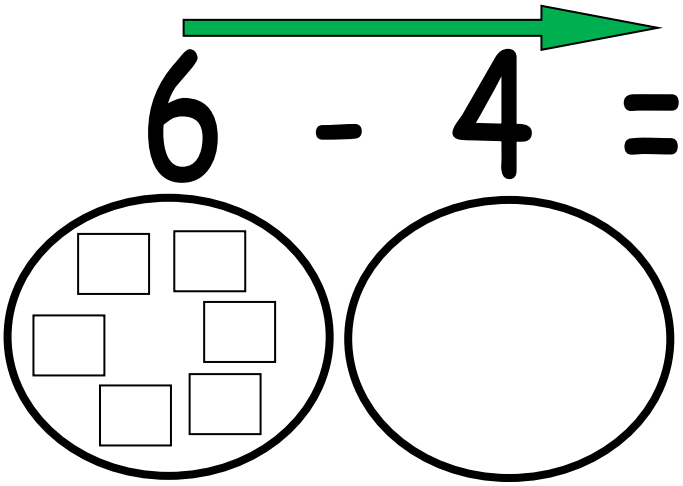
# SUBTRACTION -

(subtract, subtraction, take away, minus, decrease, leave, difference, fewer, equals, inverse)

## Reading the number sentence

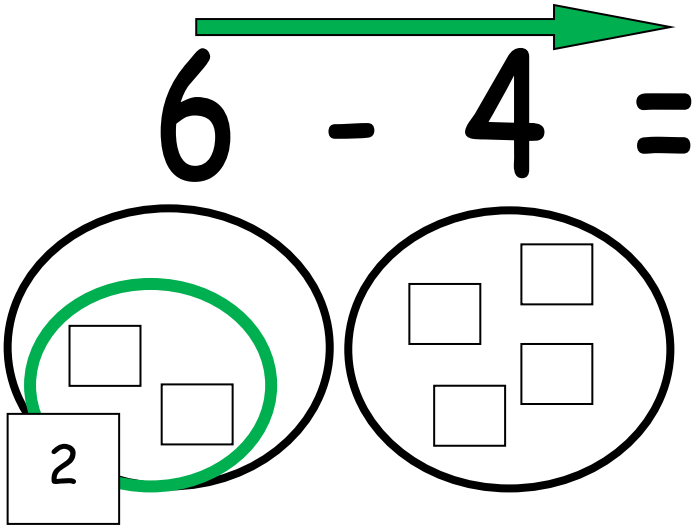



## Arranging the number sentence

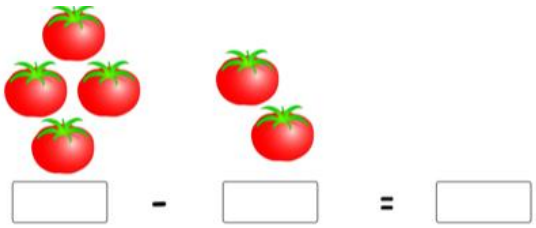


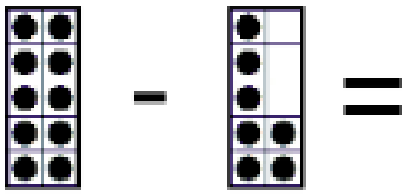

### 6 take away 4

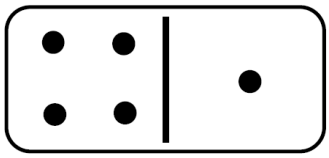
- Count how many objects you have.
- See how many need taking away.
- Count how many you are taking away.
- Check you have taken away the right amount.
- Count how many are left.



## Subtraction using objects/ pictorial representation

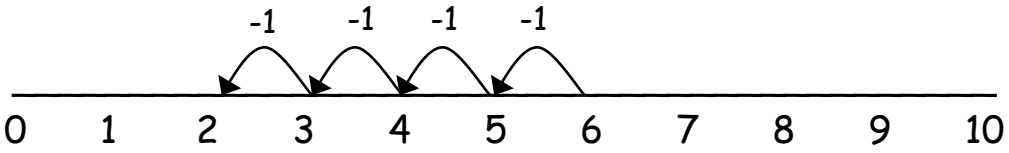




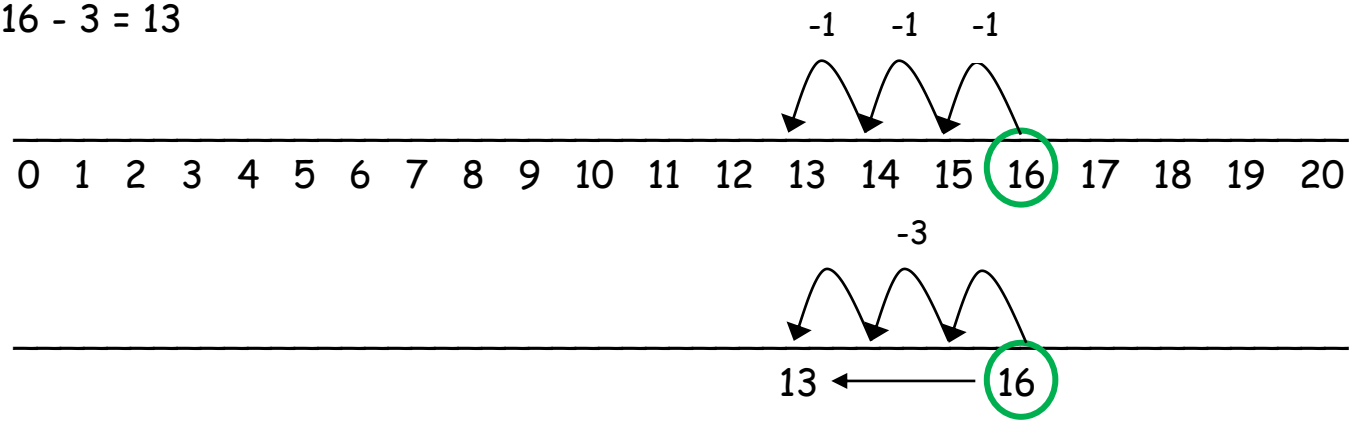


## Number Line

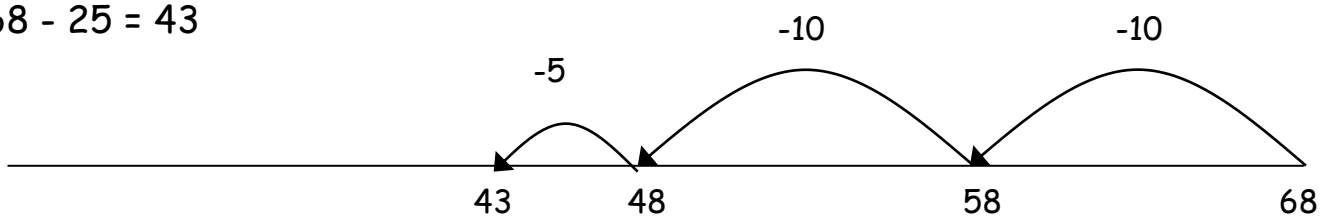
6 - 4 = 2



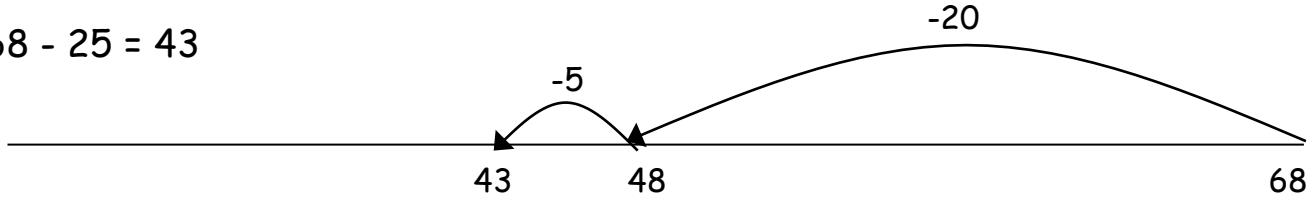

16 - 3 = 13




68 - 25 = 43

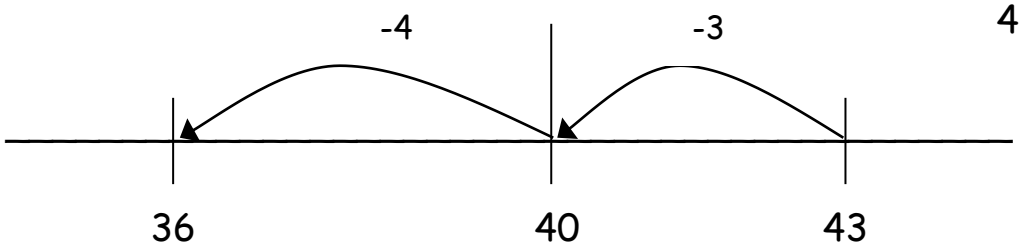



68 - 25 = 43



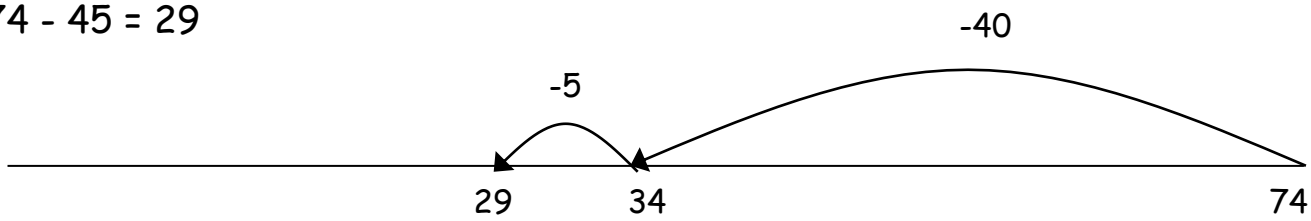
- Using bridging

43 - 7 = 36

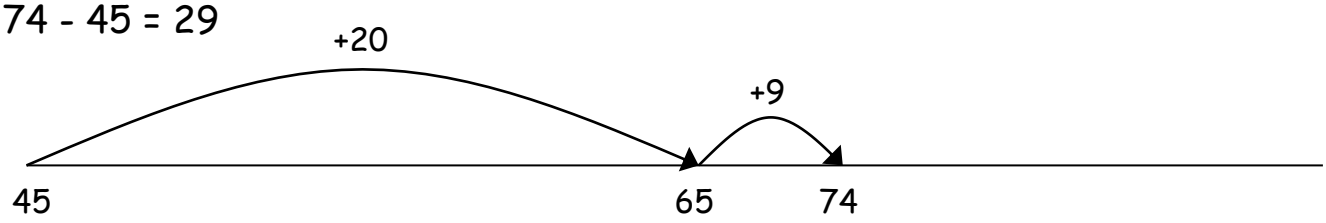



- Finding the difference by counting on

74 - 45 = 29

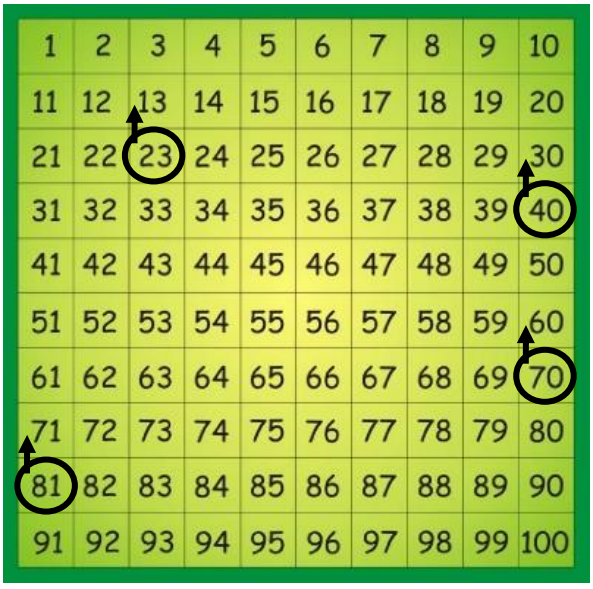


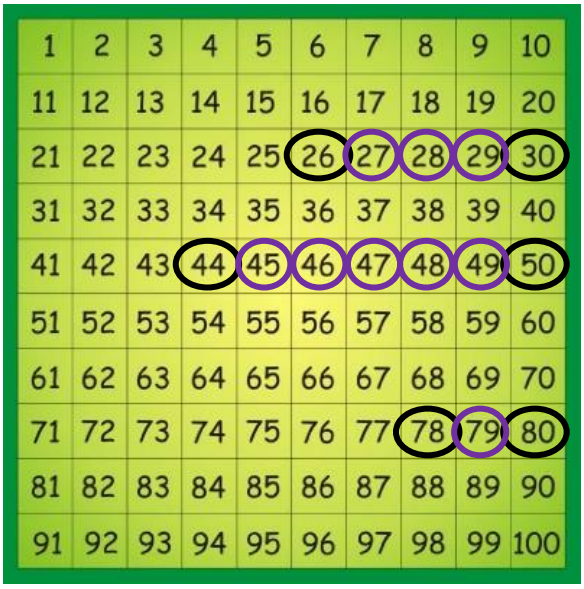

74 - 45 = 29




## Hundred Square

40 - 10 = 30  70 - 10 = 60  23 - 10 = 13  82 - 10 = 72

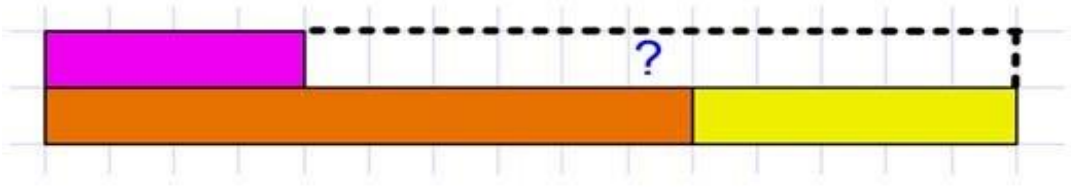



30 – 4 = 26  50 - 6 = 44  80 – 2 = 78

## Bar model

15 - 3 =

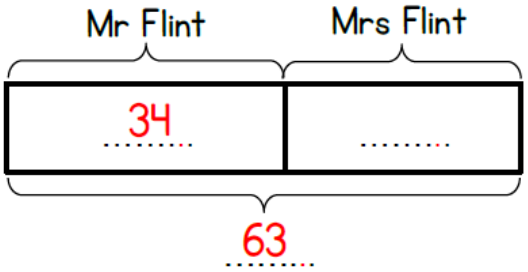



Here are Mr and Mrs Flint's cases.

Mr Flint's case weighs 34kg.

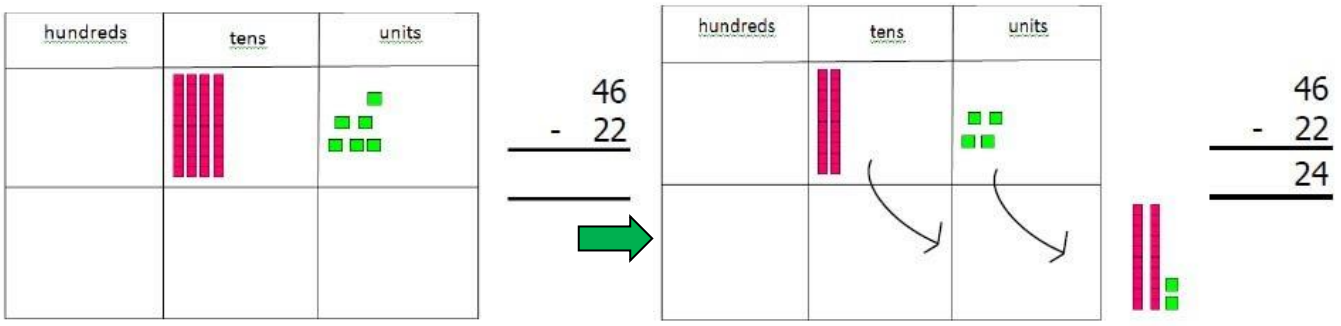
In total the cases weigh 63kg

How much does Mrs Flint's case weigh?

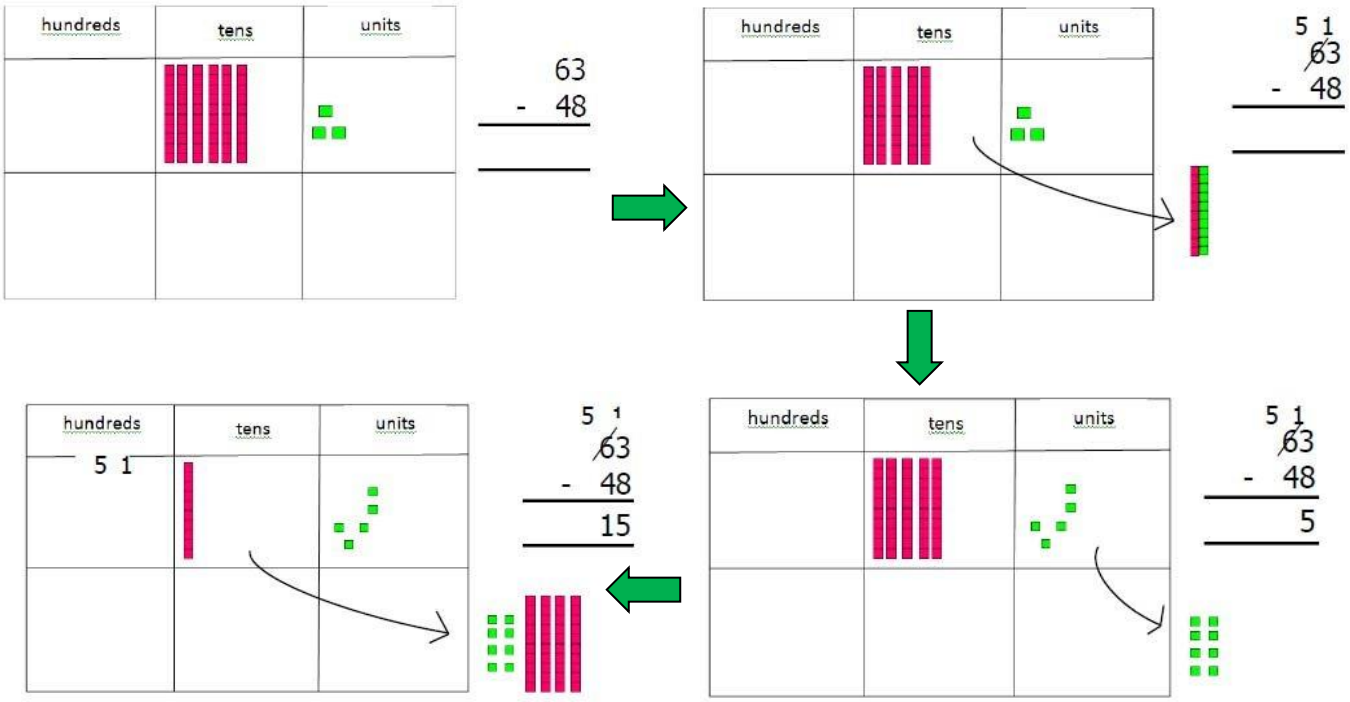



## Supported column method (subtraction)

### *Without regrouping*




### *With regrouping*



## Column method (subtraction)

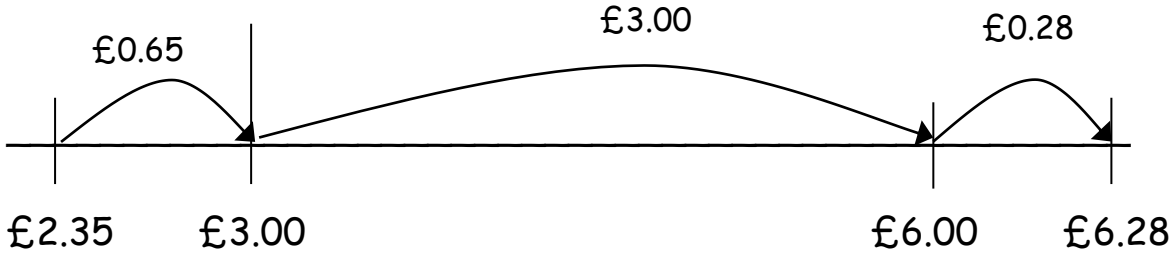
$$\begin{array}{r} 4698 \\ -\ 167 \\ \hline 4531 \end{array} \qquad \begin{array}{r} 7435^{4}\cancel{5}^{1}\cancel{8} \\ -\ 2049 \\ \hline 72309 \end{array} \qquad \begin{array}{r} \cancel{6}^{5}\cancel{4}^{1}\cancel{5}^{4}\cancel{2}^{1} \\ -\ 2615 \\ \hline 3837 \end{array} \qquad \begin{array}{r} \cancel{4}^{3}\cancel{0}^{1}\cancel{0}^{1}\cancel{5}^{1} \\ -\ 1998 \\ \hline 2007 \end{array}$$

$$\begin{array}{r} 92 \\ -\ 21 \\ \hline 71 \end{array} \qquad \begin{array}{r} \cancel{5}^{4}\cancel{6}^{1} \\ -\ 18 \\ \hline 34 \end{array} \qquad \begin{array}{r} 874 \\ -\ 523 \\ \hline 351 \end{array} \qquad \begin{array}{r} \cancel{4}^{3}\cancel{5}^{1}8 \\ -\ 265 \\ \hline 193 \end{array} \qquad \begin{array}{r} \cancel{9}^{8}\cancel{3}^{12}\cancel{2}^{1} \\ -\ 457 \\ \hline 475 \end{array}$$

## Subtracting decimals

£6.28 - £2.35 = £3.93

£3.00 + £0.65 + £0.28 = £3.93




£8.57 - £2.61 = £5.96

£5.57 + £0.39 = £5.96

$$\begin{array}{r} £9.46 \\ -\ £3.14 \\ \hline £6.32 \end{array} \qquad \begin{array}{r} \cancel{£8}^{7}.\cancel{5}^{1}7 \\ -\ £2.61 \\ \hline £5.96 \end{array} \qquad \begin{array}{r} \cancel{£3}^{2}.\cancel{6}^{15}\cancel{5}^{1} \\ -\ £1.89 \\ \hline £1.76 \end{array}$$

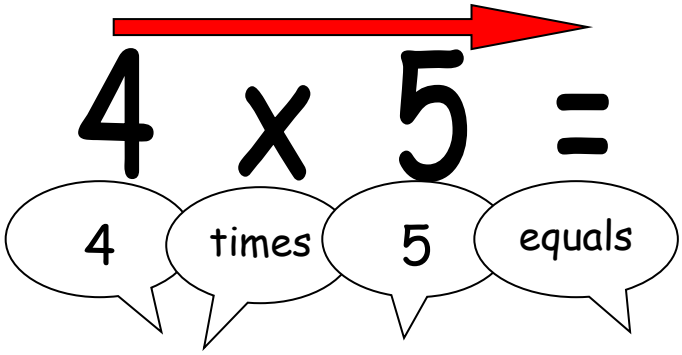
$$\begin{array}{r} 9.46 \\ -\ 3.14 \\ \hline 6.32 \end{array} \qquad \begin{array}{r} \cancel{8}^{7}.\cancel{5}^{1}7 \\ -\ 2.61 \\ \hline 5.96 \end{array} \qquad \begin{array}{r} \cancel{3}^{2}.\cancel{6}^{15}\cancel{5}^{1} \\ -\ 1.89 \\ \hline 1.76 \end{array}$$

PROGRESSION THROUGH WRITTEN CALCULATION FOR

# MULTIPLICATION X

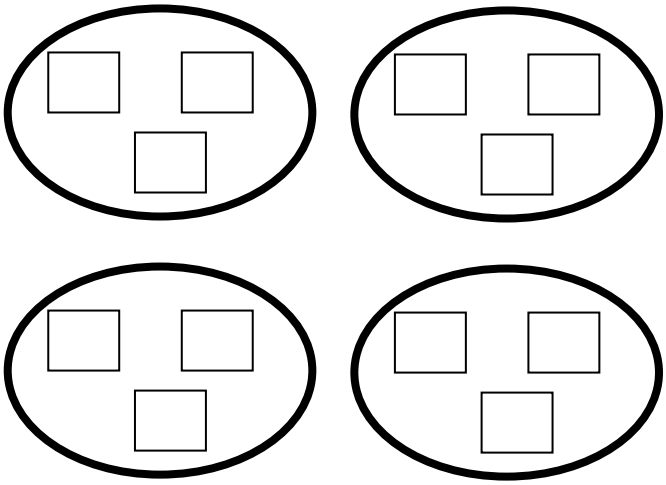
(steps, lots, groups of, times, multiply, multiplied by, repeated addition, array, product, inverse)

## Reading the number sentence

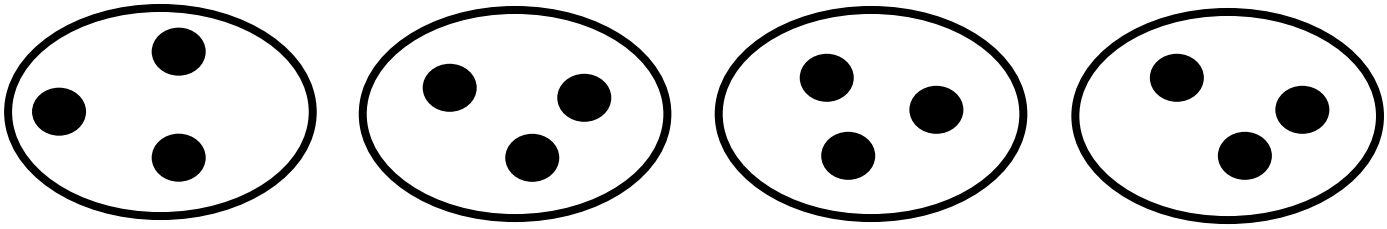



Set out 4 lots of 3 blocks

- Set out one lot of 3
- Set out another lot of 3
- Set out two more lots of 3
- Check you have 4 groups
- Check there are 3 in each group
- Count all of the objects



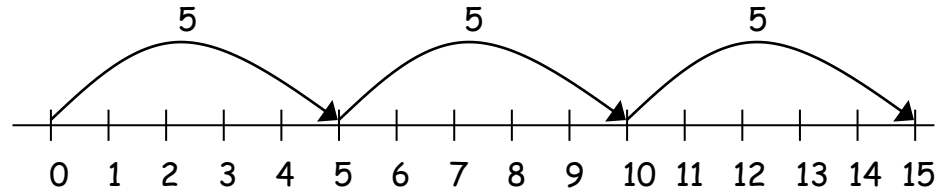
## Repeated addition

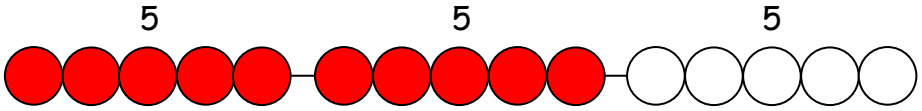


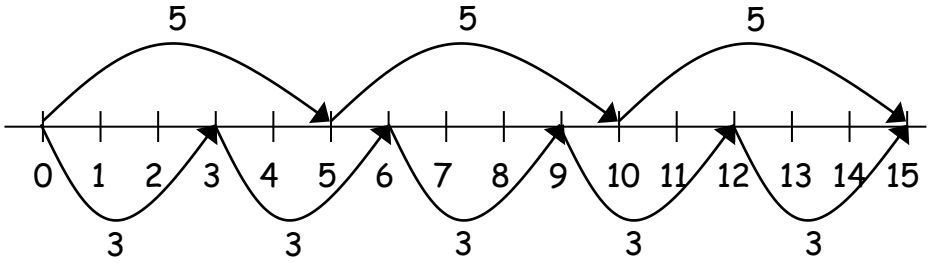
3 + 3 + 3 + 3

= 4 x 3

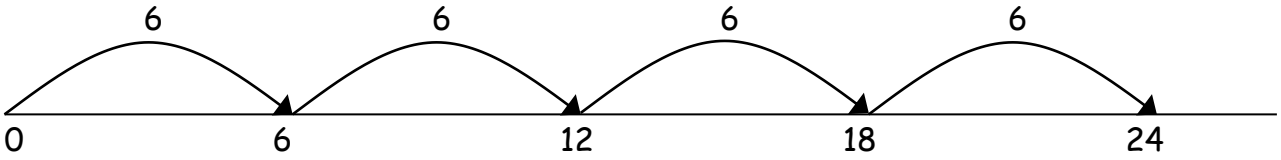
5 lots of 3 = 5 x 3 = 5 + 5 + 5

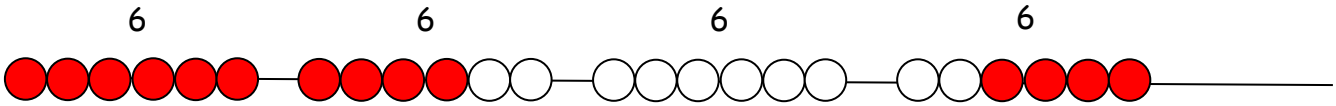


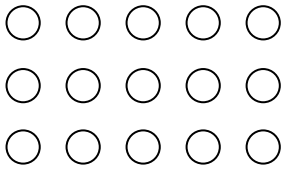




4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4



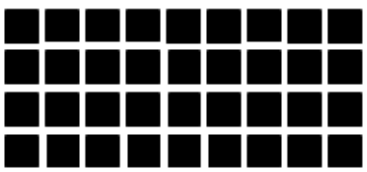




## Using arrays



5 x 3 = 15

3 x 5 = 15



9 x 4 = 36

9 x 4 = 36

- Missing digit calculations

□ x 5 = 20 3 x △ = 18 □ x ○ = 32

## Multiplying by multiples of 10 (smile multiplication)

$3 \times 50 = 150$

15

$40 \times 20 = 800$

8

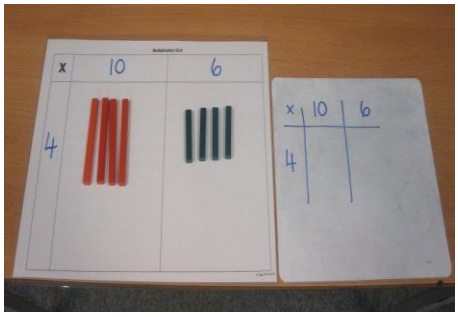
$9 \times 60 = 540$

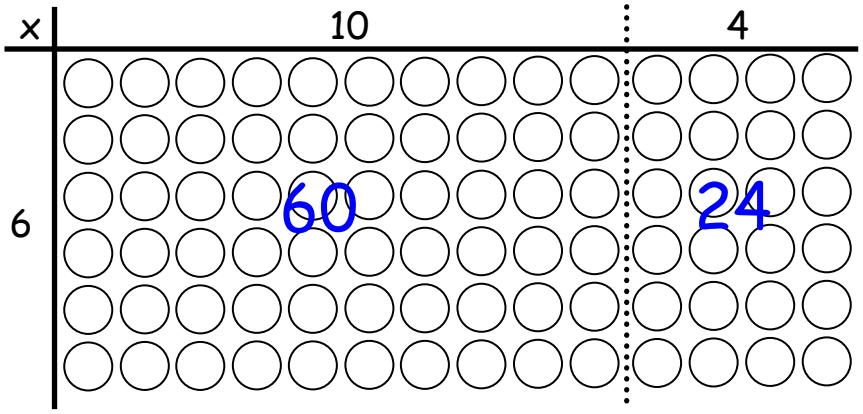
54

$80 \times 70 = 5600$

56

## Multiplying a 2 digit number by a 1 digit number




(6 x 10) + (6 x 4) ⟶ 60 + 24 ⟶ 84

$$\begin{array}{r} 14 \\ \times \quad 6 \\ \hline 84 \\ \hline {\scriptstyle 2} \end{array}$$

| x | 10 | 4 |
|---|---|---|
| 6 | 60 | 24 |

$$\begin{array}{r} 60 \\ +\ 24 \\ \hline 84 \end{array}$$

$14 \times 6$

$4 \times 6 = 24$

$10 \times 6 = 60$

$60 + 24 = 84$

80 4

## Multiplying a 3 digit number by a 1 digit number

342 x 7

| x | 300 | 40 | 2 |
|---|---|---|---|
| 7 | 2100 | 280 | 14 |

300 x 7
40 x 7
2 x 7

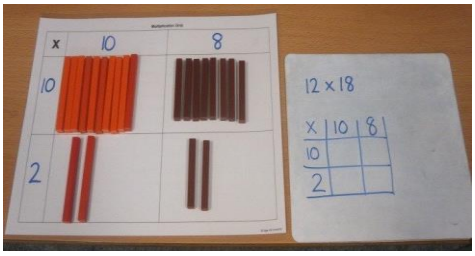
2100
+ 280
+ 14
2394

342 x 7 becomes

| | 3 | 4 | 2 |
|---|---|---|---|
| x | | | 7 |
| 2 | 3 | 9 | 4 |
| | 2 | 1 | |

Answer: 2394

## Multiplying a 2 digit number by a 2 digit number




72 x 38

70 30
2 8

| x | 70 | 2 |
|---|---|---|
| 30 | 2100 | 60 |
| 8 | 560 | 16 |

70 x 30
70 x 8
20 x 30
20 x 8

2100
+ 560
+ 60
+ 16
2736
1

| | 7 | 2 |
|---|---|---|
| x | 3 | 8 |
| | 5 | 7 | 6 |
| 2 | 1 | 6 | 0 |
| 2 | 7 | 3 | 6 |
| | 1 | | |

## Multiplying a 3 digit number by a 2 digit number

372 x 24

| x | 300 | 70 | 2 |
|---|---|---|---|
| 20 | 6000 | 1400 | 40 |
| 4 | 1200 | 280 | 8 |

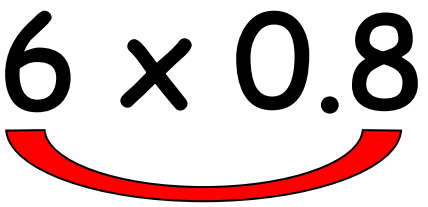
300 x 20
70 x 20
2 x 20

300 x 4
70 x 4
2 x 4

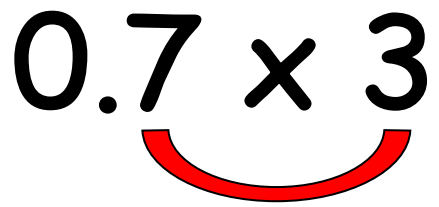
6000
+ 1400
+ 1200
+ 280
+ 40
+ 8
8928
1

| | 3 | 7 | 2 |
|---|---|---|---|
| x | | 2 | 4 |
| 1 | 4 | 8 | 8 |
| 7 | 4 | 4 | 0 |
| 8 | 9 | 2 | 8 |
| | 1 | | |

## Multiplying decimals




6 x 8 = 48  48 ÷ 10 = 4.8



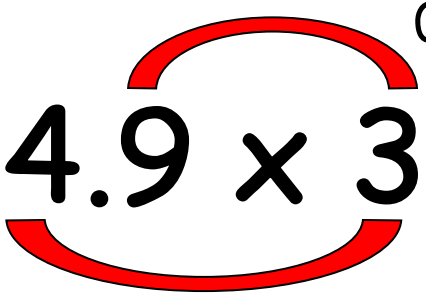

7 x 3 = 21  21 ÷ 10 = 2.1

4.9 x 3

| x | 4 | 0.9 |
|---|---|---|
| 3 | 12 | 2.7 |

4 x 3
0.9 x 3

12
+ 2.7
14.7




0.9 x 3 = 2.7

12 + 2.7 = 14.7

10 4 0.7

4 x 3 = 12

4.92 x 3

| x | 4 | 0.9 | 0.02 |
|---|---|---|---|
| 3 | 12 | 2.7 | 0.06 |

4 x 3
0.9 x 3
0.02 x 3

12
+ 2.7
+ 0.06
14.76

4.92
x 3
14.76
1 2

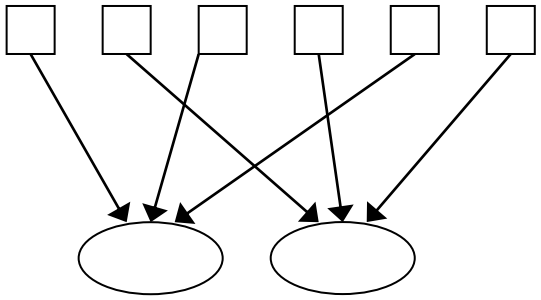
# PROGRESSION THROUGH WRITTEN CALCULATION FOR

# DIVISION ÷

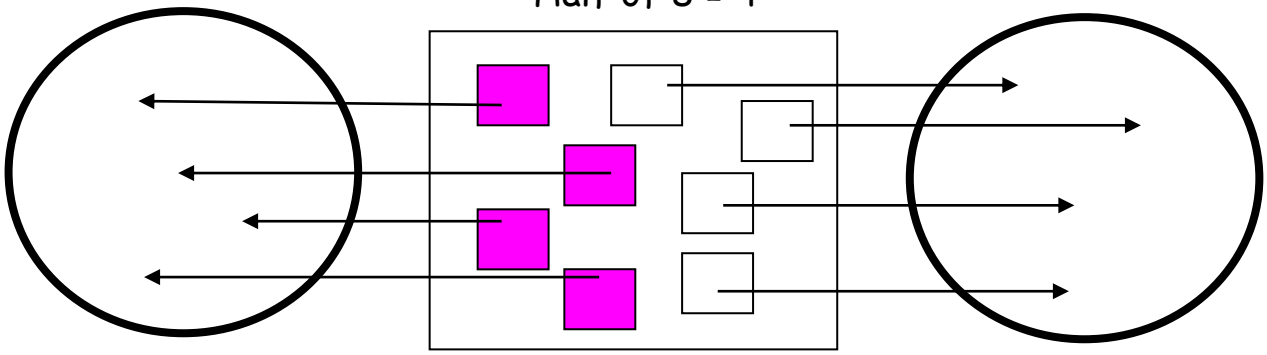
(halve, share, share equally, divide, divided by, left over, remainder, repeated subtraction, equals, inverse)

## Sharing out objects

6 sweets shared between 2 people, how many do they each get?

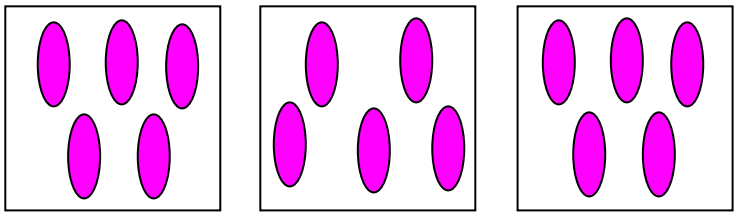


Half of 8 = 4

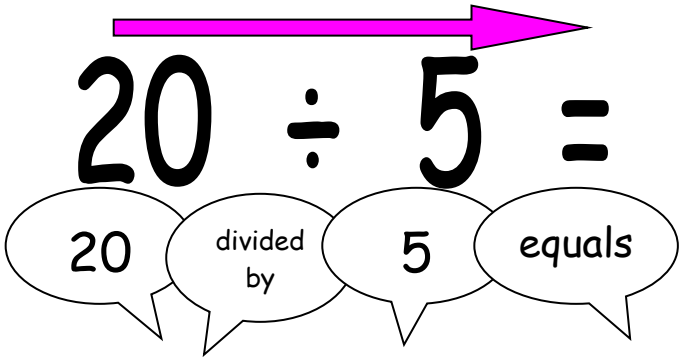


12 ÷ 3 = 4

15 ÷ 3 = 5



## Reading the number sentence



## Arranging a number sentence

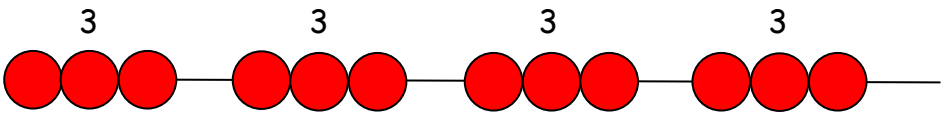
**15 ÷ 3**

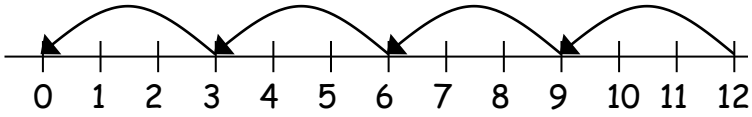
15 blocks going into piles of 3

How many lots of 3 are there in 15?

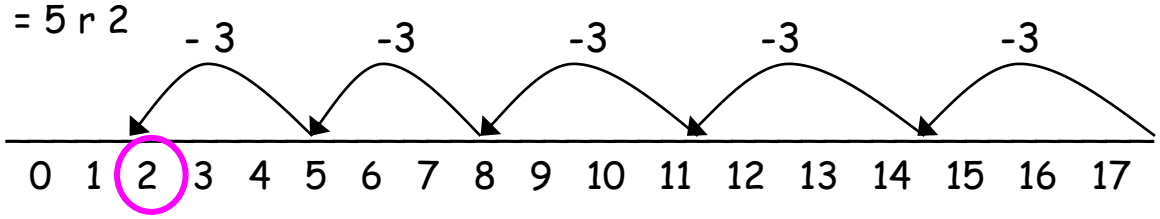
## Repeated subtraction

12 ÷ 3 = 4

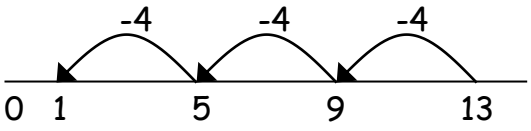



17 ÷ 3 = 5 r 2

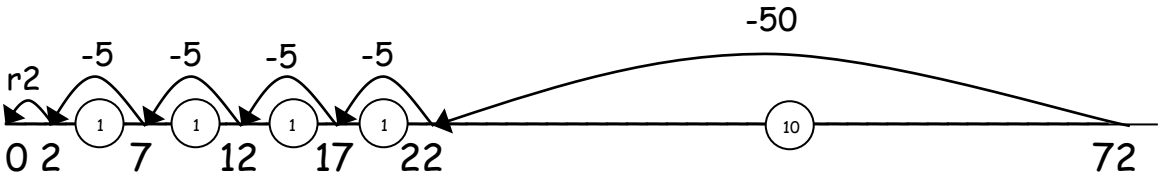



13 ÷ 4 = 3 r 1



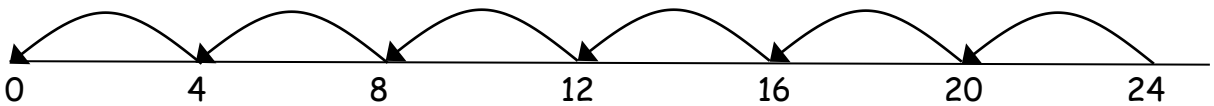

- Write down the biggest multiple you can see in the starting number. **12**
- Write how many 'lots of' that is. **3**
- Write how many are left in the starting number to show your remainder. **1**

72 ÷ 5




## Division using associated multiplication facts

24 ÷ 4 = 6 If 6 x 4 = 24 **then** 24 ÷ 4 = 6




- Missing digit calculations

□ ÷ 2 = 4 20 ÷ a = 4 □ ÷ △ = 4 24 ÷ △ = 12 y ÷ 10 = 8

## Division using smile multiplication

**150 ÷ 5**

15 ÷ 5 = 3 so 150 ÷ 5 = 30

**400 ÷ 8**

40 ÷ 8 = 5 so 400 ÷ 8 = 50

**210 ÷ 3**

21 ÷ 3 = 7 so 210 ÷ 3 = 70

## Short Division

98 ÷ 7 = 14

$$7 \overline{)\,9^{2}8}\quad 1\,4$$

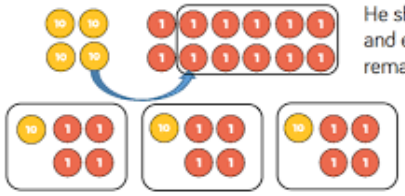
165 ÷ 5 = 33

$$5 \overline{)\,1^{1}6^{1}5}\quad 3\,3$$

152 ÷ 5 = 30 r 2

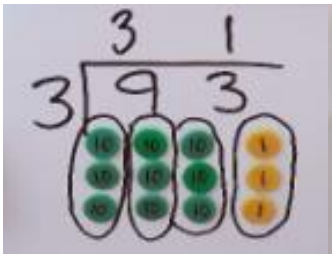
$$5 \overline{)\,1^{1}5\,2}\quad 3\,0\,r\,2$$

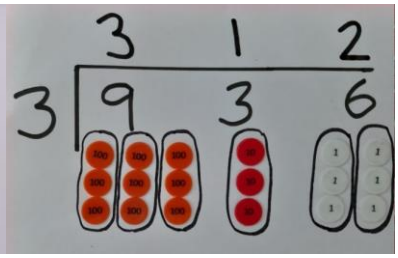
Ron uses place value counters to divide 42 into three equal groups.



He shares the tens first and exchanges the remaining ten for ones.

Then he shares the ones.
42 ÷ 3 = 14




- Recording solutions in different forms

$$6 \overline{)\,1^{1}9^{1}6}\quad 3\,2\,r\,4$$

$$6 \overline{)\,1^{1}9^{1}6}\quad 3\,2\,\tfrac{4}{6}$$

$$6 \overline{)\,1^{1}9^{1}6.{}^{4}0^{4}0}\quad 3\,2.6\,6$$

$$14 \overline{)\,2^{2}3^{9}8}\quad 1\,7$$

| 14 | 28 | 42 | 56 | 70 | 84 | 98 |
|---|---|---|---|---|---|---|
| 1x | 2x | 3x | 4x | 5x | 6x | 7x |

## Long Division (Chunking)

117 ÷ 9 = 13

$$9 \overline{)\,{}^{0}\!\not{1}{}^{1}1\,7}\quad 1\,3$$

- 90 (10x)
- 27
- 27 (3x)
- 00

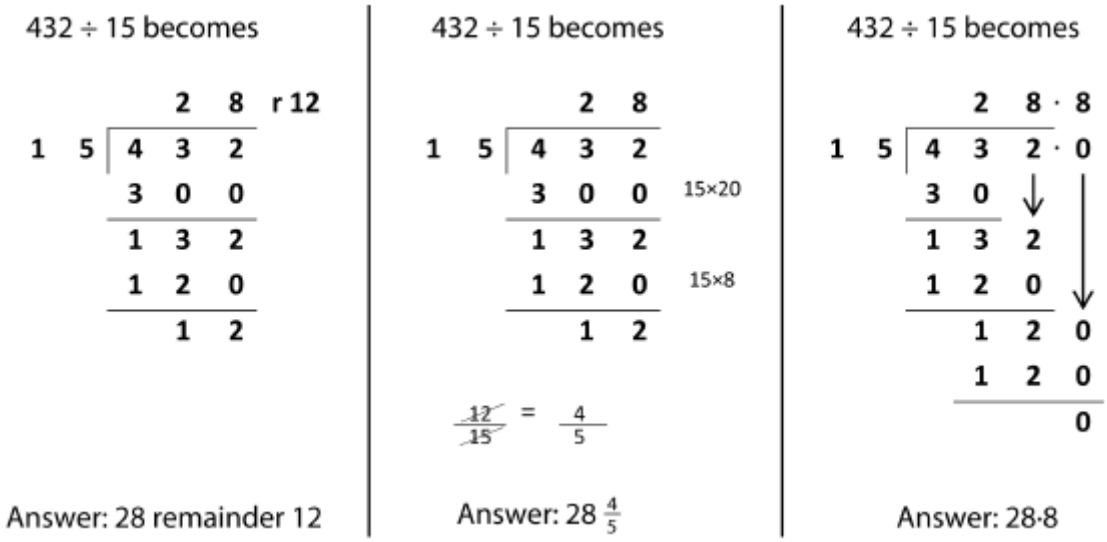
Answer : 13

89 ÷ 6 = 14 r 5

$$6 \overline{)\,89}\quad 14\,r\,5$$

- 60 (10x)
- 29
- 24 (4x)
- 05

Answer : 14 r 5




## Dividing decimals

87.5 ÷ 7

```
     12.5
7 ) 87.5
  - 70.0      10x
    17.5
  - 14.0      2x
     3.5
  -  3.5      0.5x
     0
```

Answer: 12.5

**2.4 ÷ 8**

```
      0. 3
   ------
8 | 2.4
```

If 0.3 x 8 = 2.4 **then** 2.4 ÷ 8 = 0.3

**42.4 ÷ 8**

```
      5 . 3
   ------
8 | 42.4
```

**Updated September 2023**